SN – 350

I Semester B.A./B.Sc. Examination, Nov./Dec. 2017
(2011-12 and Onwards) (N.S.) (Semester Scheme)
(Repeaters – Prior to 2014-15)
MATHEMATICS – I

ime : 3 Hours Max. Marks : 100

Instruction : *Answer **all** questions.*

(15x2=30)

I. Answer **any fifteen** questions.

1) Reduce the matrix $A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$ to echelon form.

2) Find the eigen value of the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

3) State Cayley-Hamilton Theorem.

4) Find the n^{th} derivative of $\cos^2 4x$.

5) Find the n^{th} derivative of $e^{3x} \sin 5x$.

6) If $Z = \sin x \cos y$ then verify $z_{xy} = z_{yx}$.

7) If $u = x^3 + y^3$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u$.

8) If $u = x^y$ then find $\frac{\partial^2 u}{\partial x \partial y}$.

9) If $u = x^2 + 3xy + y^2$ where $x = 2t$ and $y = t^2$ then find $\frac{du}{dt}$.

10) If $x = u(1 - v)$, $y = uv$ then find $\frac{\partial(x, y)}{\partial(u, v)}$.

11) Evaluate $\int_0^{\pi/2} \cos^8 x dx$.

P.T.O.

12) Evaluate $\int_0^1 \frac{x^3}{1+x^2}\,dx$.

13) Find 'a' such that the points (3, 2, 1) (4, a, 5), (4, 2, –2), (6, 5, –1) are coplanar.

14). Find the angle between the lines whose direction ratio's are (2, 3, 4) and (1, –2, 1).

15) Find the angle between the line $\frac{x+1}{3} = \frac{y}{1} = \frac{z-4}{2}$ and the plane $x + y + z = 6$.

16) Find the equation of the plane passing through the point (–1, 2, –3) and parallel to the plane $2x + y + z + 4 = 0$.

17) Find K such that $\frac{x-3}{1} = \frac{y-2}{3} = \frac{z-4}{4}$ and $\frac{x-4}{2} = \frac{y-2}{3} = \frac{z-2}{K}$ are coplanar.

18) Find the equation of the sphere with center (5, –2, 3) and radius 5 units.

19) Find the equation of the right circular cone which has its axis along the Y-axis, vertex at the origin and semi vertical angle is 30°.

20) Find the equation of right circular cylinder of radius 2 and whose axis is

$$\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-3}{5}.$$

II. Answer **any two** questions. **(2×5=10)**

1) Reduce to normal and hence find its rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$.

2) Show that the equations. $x + y + z = 6$, $x + 2y + 3z = 14$, $x + 4y + 7z = 30$ are consistent and hence solve.

3) Using Cayley-Hamilton Theorem find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$.

4) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

III. Answer **any four** questions. **(4×5=20)**

1) Find the n^{th} derivative of $\frac{2x}{(x-3)^2(x+1)}$.

2) If $x = \sin t$ and $y = \cos pt$ then prove that
$(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - (n^2 - p^2)\, y_n = 0$.

3) If $u = \cos^{-1}\left(\frac{x^3 + y^3}{x + y}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -2 \cot u$.

4) State and prove Euler's theorem for homogenous function of two variables.

5) If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$ then prove that $6\frac{\partial u}{\partial x} + 4\frac{\partial u}{\partial y} + 3\frac{\partial u}{\partial z} = 0$.

6) If $x = r\cos\theta$, $y = r\sin\theta$, then find $J = \frac{\partial(x,y)}{\partial(r,\theta)}$ and $J' = \frac{\partial(r,\theta)}{\partial(x,y)}$. Also verify $JJ' = 1$.

IV. Answer **any two** questions. **(2×5=10)**

1) Obtain the reduction formula for $\int \sin^n x\, dx$, where n is a positive integer.

2) Evaluate $\int_0^{\pi} x \sin^6 x\, dx$.

3) Using the Lebnitz rule of differentiation under integral sign show that
$\int_0^{\pi} \frac{\log(1 + 2\cos x)}{\cos x}\, dx = \pi \sin^{-1} \alpha$.

V. Answer **any four** questions. **(4×5=20)**

1) Show that the two lines whose direction cosines satisfy the equations $l + 2m + 3n = 0$ and $mn - 4nl + 3lm = 0$ are at right angles.

2) Find the value of 'a' such that the points A (3, 2, 1), B (4, a, 5), C(4, 2, –2) and D (6, 5, –1) are coplanar.

3) If a line makes angles α, β, γ and δ with four diagonals of a cube, show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.

4) Find the image of the point (1, 2, 3) in the plane $x + y + z = 9$.

5) Find the shortest distance between the lines $\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-2}{-2}$ and $\frac{x-2}{2} = \frac{y-8}{2} = \frac{z+1}{-1}$.

6) Find the equation of the plane which contains the two parallel lines $\frac{x-3}{3} = \frac{y+4}{2} = \frac{z-1}{1}$ and $\frac{x+1}{3} = \frac{y-2}{2} = \frac{z}{1}$.

VI. Answer **any two** questions. (2×5=10)

1) Find the equation of the sphere which passes through the points (1, 2, 3) (0, 3, 3), (1, 3, 2) and having center on the plane $x + 4y + z = 0$.

2) Find the equation of the right circular cone whose vertex is the origin, whose axis is on the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and which has semi vertical angle is 30°

3) Find the equation of the right circular cylinder of radius 3 and axis $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$.